1) Typo: b should be $\frac{1}{\tau}$

First, we need the energy.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} e^{-2t/\tau} dt = \left. \frac{e^{-2t/\tau}}{-2/\tau} \right]_{t=0}^{\infty} = \frac{\tau}{2} e^0 = \frac{\tau}{2}$$

The energy in the frequencies less than W is:

$$E(W) = \int_{-W}^{W} |X(f)|^2 df.$$

$$e^{-t} u(t) \xrightarrow{\mathcal{F}} \frac{1}{1+2\pi j f} \quad , \quad e^{-t/\tau} u(t) \xrightarrow{\mathcal{F}} \tau \frac{1}{1+2\pi j f \tau}$$

$$|X(f)|^2 = X(f) X^*(f) = \frac{\tau}{1+2\pi j f\tau} \frac{\tau}{1-2\pi j f\tau} = \frac{\tau^2}{1+4\pi^2 f^2 \tau^2}$$

$$\int \frac{1}{1+f^2} df = \tan^{-1}(f) \rightarrow \int \frac{\tau^2}{1+(2\pi\tau f)^2} df = \frac{\tau^2}{2\pi\tau} \tan^{-1}(2\pi\tau f)$$

$$\int_{-W}^{W} |X(f)|^2 df = \underbrace{2 \int_0^W |X(f)|^2 df}_{\text{(even)}} = \frac{\tau}{\pi} \tan^{-1}(2\pi\tau W) - 0$$

$$W = \frac{1}{2\pi\tau} \rightarrow E(W) = \frac{\tau}{\pi} \tan^{-1}\left(\frac{2\pi\tau}{2\pi\tau}\right) = \frac{\tau}{\pi} \tan^{-1}(1) = \frac{\tau}{\pi}\frac{\pi}{4} = \frac{\tau}{4}$$

fraction: $\boxed{\frac{1}{2}}$

$$W = \frac{2}{\pi\tau} \rightarrow E\left(\frac{2}{\pi\tau}\right) = \frac{\tau}{\pi} \tan^{-1}\left(2\pi\tau \cdot \frac{2}{\pi\tau}\right) = \frac{\tau}{\pi} \tan^{-1}(4) = 0.4220\,\tau$$

fraction: $\boxed{0.8440}$

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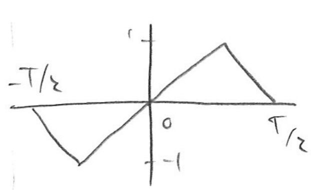
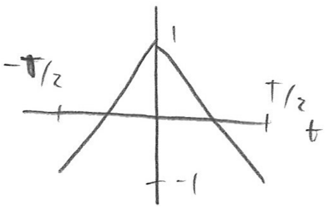
2)



The problem is a bit ambiguous, but part of the point is to know what a triangle wave is and set up the solution in a reasonable way.

continued...

2. cont. Method 1: Poisson sum



$x(t)$ = triangle wave

$g(t)$ = one period, $-T/2 < t < T/2$

$$x(t) = \sum_{n=-\infty}^{\infty} g(t-nT) \xrightarrow{\mathcal{F}} X[k] = \frac{1}{T} G\left(\frac{k}{T}\right)$$

$$g(t) = 2\Lambda\left(\frac{t}{T/2}\right) - \Pi\left(\frac{t}{T}\right)$$

$$G(f) = 2\,\frac{T}{2}\,\mathrm{sinc}^2\left(\frac{T}{2}f\right) - T\,\mathrm{sinc}(Tf)$$

$$X[k] = \frac{1}{T} G\left(\frac{k}{T}\right) = \mathrm{sinc}^2\left(\frac{k}{2}\right) - \mathrm{sinc}(k)$$

Recall that $\mathrm{sinc}(k) = \delta[k]$ and $\mathrm{sinc}\left(\frac{k}{2}\right) = \begin{cases} -\frac{(-1)^k}{\pi k}, & k \text{ odd} \\ 0, & k \text{ even} \\ 1, & k = 0 \end{cases}$

$$\boxed{X[k] = \begin{cases} \left(\frac{2}{\pi k}\right)^2 & k \text{ odd} \\ 0 & k \text{ even} \end{cases}}$$

Method 2: direct integration

$$X[k] = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-2\pi j k t/T}\, dt \qquad x(t) = x(-t)$$

$$\int_{-T}^{T} \text{even} \cdot \text{even}\, dt = 2\int_0^T \cdots dt$$

$$= \frac{2}{T}\int_0^{T/2} \left(1 - \frac{4}{T}t\right)\cos(2\pi t/T)\, dt \qquad \int_{-T}^{T} \text{even} \cdot \text{odd}\, dt = 0$$

tedious integration yields:

$$X[k] = -\frac{2\cos(\pi k) + k\pi\sin(\pi k) - 2}{\pi^2 k^2}$$

$$= -\frac{2(-1)^k + 0 - 2}{\pi^2 k^2} = \frac{2}{\pi^2 k^2}\left(1 - (-1)^k\right) = \boxed{\begin{cases} \frac{2^2}{\pi^2 k^2} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}}$$

Finally, $X[0] = 0$ by inspection (equal area above and below x-axis).

3.

$$x(t) = A\sin(2\pi t/\tau)\,\Pi(t/\tau)$$

$$= \frac{A}{2j}\left(e^{2\pi j t/\tau} - e^{2\pi j t/\tau}\right)\Pi(t/\tau)$$

Let's use the multiplication rule

$$e^{2\pi j t/\tau} \xrightarrow{\mathcal{F}} \delta\left(f - \tfrac{1}{\tau}\right)$$

$$\Pi\left(\tfrac{t}{\tau}\right) \xrightarrow{\mathcal{F}} \tau\,\mathrm{sinc}(\tau f)$$

$$x(t) \xrightarrow{\mathcal{F}} \frac{A}{2j}\left(\delta\left(f - \tfrac{1}{\tau}\right) - \delta\left(f + \tfrac{1}{\tau}\right)\right) * \tau\,\mathrm{sinc}(\tau f)$$

$$\delta(f - f_0) * G(f) = G(f - f_0)$$

$$X(f) = \frac{A}{2j}\tau\left(\mathrm{sinc}\left(\tau\left(f - \tfrac{1}{\tau}\right)\right) - \mathrm{sinc}\left(\tau\left(f + \tfrac{1}{\tau}\right)\right)\right)$$

$$= \boxed{\frac{A}{2j}\tau\left(\mathrm{sinc}(\tau f - 1) - \mathrm{sinc}(\tau f + 1)\right)}$$

3.b

The above can be further simplified:

$$\mathrm{sinc}(\lambda - 1) - \mathrm{sinc}(\lambda + 1) = \frac{1}{\pi}\left(\frac{\sin(\pi\lambda - \pi)}{\lambda - 1} - \frac{\sin(\pi\lambda + \pi)}{\lambda + 1}\right)$$

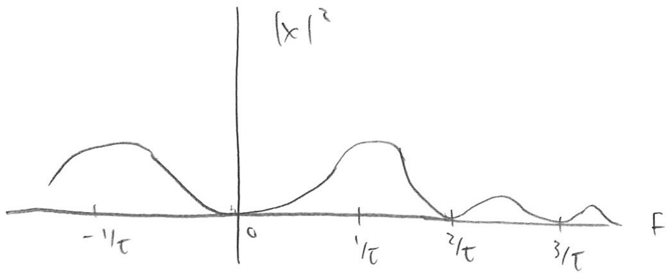
$$= \frac{1}{\pi}\left(\frac{\cos(\pi\lambda)\cancelto{0}{\sin(-\pi)} + \cancelto{1}{\cos(-\pi)}\sin(\pi\lambda)}{\lambda - 1} - \frac{\cos(\pi\lambda)\cancelto{0}{\sin(\pi)} + \cancelto{1}{\cos(\pi)}\sin(\pi\lambda)}{\lambda + 1}\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{\lambda - 1} - \frac{1}{\lambda + 1}\right)\sin(\pi\lambda) = \frac{1}{\pi}\sin(\pi\lambda)\left(\frac{(\lambda+1) - (\lambda-1)}{(\lambda-1)(\lambda+1)}\right) =$$

$$= \frac{2}{\pi}\,\frac{\sin(\pi\lambda)}{\lambda^2 - 1}$$

$$X(f) = \frac{A\tau}{\pi j}\cdot\frac{\sin(\pi\tau f)}{(\tau f)^2 - 1} \qquad |X(f)|^2 = \frac{A^2\tau^2}{\pi^2}\,\frac{\sin^2(\pi\tau f)}{((\tau f)^2 - 1)^2}$$

3.b cont.



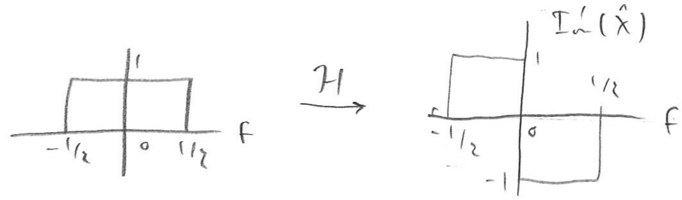
4.

$x(t) = \text{sinc}(t)$

$R_{x,x}(\tau) = \mathcal{F}^{-1}\left\{|X(f)|^2\right\} = \mathcal{F}^{-1}\left\{|\Pi(f)|^2\right\} = \mathcal{F}^{-1}\left\{\Pi(f)\right\} = \boxed{\text{sinc}(f)}$

5.

$x(t) = \text{sinc}(t)$



$X(f) = \Pi(f)$

$\hat{X}(f) = -j\,\text{sgn}(f)\,\Pi(f) = -j\,\Pi\left(\frac{f-1/4}{1/2}\right) + j\,\Pi\left(\frac{f+1/4}{1/2}\right)$

$\Pi\left(\frac{f}{1/2}\right) = \Pi(2f) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2}\,\text{sinc}\left(\frac{t}{2}\right)$

$\Pi\left(\frac{f \pm 1/4}{1/2}\right) \xrightarrow{\mathcal{F}^{-1}} \frac{1}{2}\,\text{sinc}\left(\frac{t}{2}\right) \cdot e^{\mp 2\pi j t/4}$

$\hat{x}(t) = -\frac{j}{2}\,\text{sinc}\left(\frac{t}{2}\right)\left(e^{2\pi j t/4} - e^{-2\pi j t/4}\right)$

$= \boxed{+\,\text{sinc}\left(\frac{t}{2}\right)\sin(2\pi t/4)}$ Apparently, this simplifies to $\frac{1-\cos(\pi t)}{\pi t}$

6.

$x(t) = A(1+\mu m(t))\cos(2\pi f_c t) = A\cos(2\pi f_c t) + A\mu m(t)\cos(2\pi f_c t)$

$PEP = \frac{A^2}{2} + \frac{A^2\mu^2 m_p^2}{2}$, $P_{carrier} = \frac{A^2}{2}$, $P_{signal} \approx \frac{A^2\mu^2 m_{rms}^2}{2}$

$m_{rms} = \frac{m_p}{\sqrt{2}} \rightarrow P_{signal} = \frac{A^2\mu^2 m_p^2}{4}$

6 cont.

$$P_T = P_c + 2P_s = \frac{1}{2}A^2(1+\mu^2 m_p^2) = \frac{1}{4}PEP = \frac{1}{4}\frac{1}{2}A^2(1+\mu^2 m_p^2)$$

It appears that the relationship is the same for all μ, so the maximum $\mu = 1$.

7)

A PSK signal has constant envelope, because it's a type of phase modulation.

8) I forgot to say that $540 < f_{RF} < 1600$ kHz.

$f_{LO} = f_c + f_{IF} \rightarrow 995 < f_{LO} < 2055$

$C = \left(\frac{2\pi}{f_{LO}}\right)^2 \frac{1}{L} \rightarrow \boxed{40\,\mu F > C > 9.35\,\mu F}$ this range is clearly more reasonable to realize.

$f_{LO} = f_c - f_{IF} \rightarrow \boxed{5.46\,mF > C > 30.1\,\mu F}$